A New Computational Framework for Fatigue Crack Growth Analysis of Components

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1. Introduction

Two contrasting methods are traditionally available to analyze fatigue crack growth (FCG) lifetime in components. On the one hand, engineering software such as NASGRO and AFGROW provides pre-programmed stress intensity factor (SIF) solutions for simplified crack and component geometries, sophisticated crack growth equations including load interaction models, and relatively fast execution times—often only a few seconds. However, the SIF solutions in these codes are often for simple uniform or linear stress distributions, and the user is left with the task of interpreting how best to (manually) transfer dimensions and stresses from component models into the simplified fracture models.

On the other hand, numerical fracture software such as FRANC or BEASY offers integrated modeling of the crack in the component, calculating the SIF more accurately with finite element (FE) or boundary element (BE) methods based on the actual three-dimensional (3D) crack and component geometries and stresses, and updating the mesh as the crack grows. However, these codes generally offer a limited menu of crack growth models for complex variable amplitude loading, and their execution times for large load spectra can be extremely long in some cases—perhaps several hours.

Neither of these classes of FCG analysis methods is typically integrated directly with reliability assessment to determine the probability of fracture. Some external linkage with an independent computer program is usually required to perform this calculation, and this introduces further inefficiencies into the analysis process.

In this paper, a new framework for FCG analysis of components that offers a unique balance of accuracy and efficiency, fully integrated with reliability assessment, is described. New weight function SIF solutions with high accuracy and efficiency accommodate complex stress gradients throughout the crack plane. A sophisticated graphical user interface (GUI) provides a direct interface with 2D/3D FE models to extract and visualize geometry and stress information. The GUI can also build an optimum fracture mechanics model automatically with little user intervention. Advanced FCG algorithms are integrated directly with probabilistic algorithms to calculate reliability. Ultimately, this framework will facilitate automated reliability calculations addressing all potential crack locations in the component.

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2. New Weight Function Stress Intensity Factor Solutions

Pre-programmed SIF solutions in FCG software are often formulated in terms of uniform remote loading (tension or bend). Some weight function (WF) solutions have been available to address arbitrary stress gradients on the crack plane in the corresponding uncracked body. However, these WF solutions are almost always limited to stresses varying along one direction only, and the solutions themselves are sometimes based on approximate formulations with accuracy limitations.

A new family of WF SIF solutions with improved accuracy and numerical efficiency is under development [1-3]. New solutions are now available for corner, surface, and embedded cracks with elliptical shapes in plates and at holes. Solutions for univariant stress gradients employ the approximate WF form proposed by Glinka [4]. Independent solutions are derived at the surface and maximum depth positions for surface and corner cracks, and for the major and minor axis intersections for elliptical embedded cracks. For the crack tip at the maximum depth (c-tip) position, for example, the weight function is

\[
W_a = \frac{2}{\sqrt{\pi}} \left[ 1 + M_{1a} \sqrt{\frac{x}{c}} + M_{2a} \cdot \frac{x}{c} + M_{3a} \left( \frac{x}{c} \right)^2 \right]
\]

where the \(M\) parameters are functions of the crack shape and normalized reference SIF solutions for uniform tension or a linear stresses applied to the crack surface.

These reference solutions are highly accurate SIF calculations generated using FADD3D [5], a BE code for 3D fracture analysis that includes a novel crack-tip element permitting the computation of the SIF directly from the nodal information along the crack front. In some cases, a special hybrid FADD-FE code [6] was used that models the region away from the crack with finite elements and details only the region around the crack with boundary elements. This approach provides improved computational efficiency and numerical stability, especially for extreme geometries. A large number of reference solutions were developed for each crack type covering a wide range of crack sizes and shapes.

The most significant advance in this new family of SIF solutions is a unique bivariant formulation that accommodates arbitrary stress gradients in all directions on the crack plane. This new formulation [2, 3] is based on the point weight function proposed by Orynyak [7] for an elliptical crack in an infinite body, with additional terms to account for free boundary corrections and finite width and thickness effects, all calibrated by three reference SIF solutions.

Because the WF solutions require integration of the weight functions and the stress gradients over the crack surface, they can be computationally intensive (slow), and this is especially true for the bivariant solutions that require two-dimensional integration. In order to improve the computational efficiency so that
the solutions would be practical for routine engineering use, two numerical algorithms were developed [3]. The first algorithm replaces the real-time integration with a closed-form double series summation based on integrands that were calculated in advance and stored in tabular form. The second algorithm generates a table of complete SIF solutions in advance for specific geometric combinations relevant to the initial conditions of the specific fracture model, updating the table as needed when the crack grows beyond the original table limits. The combined benefit of these two algorithms can be greater than two orders of magnitude in speed with no significant loss of accuracy.

These new solutions have been implemented in both the NASGRO and DARWIN computer codes. Analytical validation against independent FADD and FE has been performed, and experimental validation is underway.

3. Direct Interface Between Finite Element and Fracture Models

The advantage of the new univariant and bivariant SIF solutions is that they permit the analyst to address the actual complex stress variations that may exist in a component. Current design practices routinely employ 2D and 3D FE analysis to calculate these stresses accurately. However, in order to perform traditional FCG analysis using software like NASGRO or AFGROW, the analyst has to translate the FE models and stresses into simplified fracture models that are generally based on rectangular plates with simple stress gradients. This translation involves not only the judgment of how best to represent complex shapes and stresses with simple shapes and stresses, but also the burden of extracting dimensions and stresses from the FE models and then introducing them to the fracture models in different formats. This process is not only time-consuming, but also subject to error.

A new approach [8] has been developed for the new computational framework where the finite element geometry model and stress results files are read directly into the fracture software. Filtering schemes in the model translator facilitate extraction of submodels from larger assembly models. The graphical user interface (GUI) in the fracture software facilitates visualization of the model geometry and results. The analyst can then use simple mouse functions to locate a crack on the FE model and to superimpose the fracture geometry model (e.g., a rectangular plate width and thickness). The dimensions and stresses are then automatically extracted from the FE model and fed to the fracture model. For 2D axisymmetric models with dominant hoop stresses, these operations are carried out directly on the component cross-section, and the initial crack can be located anywhere in the component volume. Figure 1 shows a surface crack model on the hoop plane of an axisymmetric component.

For general 3D models, the analyst first locates the initial crack on the component surface. The most likely crack plane (the plane of maximum principal stress passing through the crack location) is determined, and the 3D model is sliced
along this plane to generate a 2D model. The analyst can then construct the idealized fracture model on this slice plane. Again, the stresses and dimensions required to feed the univariant or bivariant WF SIF solution are automatically extracted. Figure 2 [2] illustrates this process.

4. Automatic Fracture Model Generation

The novel GUI functionality described in the last section permits the analyst to quickly construct fracture models from the FE model. However, the quality of the resulting fracture model still depends on the skill and judgment of the analyst, which can be problematic if he or she has limited experience or formal training in fracture mechanics. Furthermore, integrity analysis of components in which fatigue cracks can form at material anomalies located anywhere in the volume of the component may require the construction of large numbers of fracture models. Even with the GUI tools, this can still be a time-consuming (expensive) process.

An alternative scheme is currently under development that will automatically determine (without user input) the orientation, size, and stress input for a fracture model that will produce accurate life results, given only the 2D model (or slice) and the initial crack location. A first-generation algorithm of this type was recently developed by Emery [9] as part of a larger effort to develop a multiscale damage and durability simulation methodology. However, the Emery auto-modeling was limited to simplified stress gradients and to semi-infinite geometry models that neglected some boundary effects.

The new automatic fracture model generation algorithm emulates the judgment of an experienced user by orienting and sizing a rectangular plate fracture model to reflect the actual component boundaries in the vicinity of a surface, corner, or embedded crack. Special algorithms accommodate curved boundaries and non-normal corners. Plate models for embedded cracks near external boundaries are oriented to accommodate automatic transition to surface cracks. Embedded plate models are otherwise oriented to capture the most significant univariant stress gradient near the crack. The final plate model is not always fully contained within the component boundaries, but plate width and thickness are sized to preserve appropriate ligaments along the primary axes of the crack, and to prevent the crack itself from going outside the actual component boundaries. The algorithm estimates the critical crack size as an aid to making some sizing decisions, but requires no fatigue crack growth calculations, and so a large number of fracture models can be constructed in very little computational time.

Figure 3 illustrates some aspects of the algorithm for an embedded crack near a boundary in an arbitrary body. The plate orientation is defined here by the minimum distance from the crack to the boundary, and trial plate dimensions are defined by intersections of the axes with the boundary. Some plate dimensions are updated to reflect the minimum distance from the crack to the plate boundary in each half of each quadrant. Figure 4 shows the boundaries of the fracture
Figure 1. 2D GUI showing crack, fracture mechanics plate model, and univariant stress gradient superimposed on finite element model.

Figure 2. Illustration of methodology using 3D FE results: (a) import 3D FE model and locate surface crack, (b) determine principal stress plane, (c) slice model to reveal 2D crack growth plane, and (d) define fracture mechanics model.
Figure 3. Schematic illustration of automatic fracture model algorithm for embedded crack near a boundary in an arbitrarily shaped body.

Figure 4. Six sample fracture models generated by the automatic fracture model algorithm: (a) the FE model given as input; (b) generated fracture models – two (pink and red) corner crack models, two (green and cyan) surface crack models and two (blue and dark gray) embedded crack models. The circles designate the estimated critical crack sizes based on the univariant stress gradient near the crack center.
model plate generated by the algorithm for an assortment of initial crack locations in an axisymmetric component with arbitrary cross-sectional shape and a complex distribution of hoop stresses.

5. Integrated Probabilistic Analysis

Engineering design and life management approaches are increasingly requiring some quantification of the component reliability—the probability of failure—rather than simply relying on some historical safety factor with ambiguous implications. Reliability calculations require a probabilistic analysis of the fracture problem in which single values of the input variables are replaced with statistical distribution functions and the result is expressed in terms of the probability of failure occurring at different numbers of elapsed fatigue cycles. This requires some integration of the FCG calculation and the probabilistic calculation. Historically, this integration has been performed by linking fracture software with independent probabilistic analysis software, such as NESSUS (a general-purpose probabilistic analysis package) or PROF (a tailored code for FCG reliability). However, this manual integration introduces inherent computational inefficiencies (which can be fatal for probabilistic computations requiring very large numbers of Monte Carlo simulations) and generally reduces flexibility.

In the new computational framework, tailored probabilistic analysis is integrated directly with the fatigue crack growth life calculations in the same software [8]. The probabilistic algorithms currently permit five primary random variables: the initial crack size; a life scatter factor that represents randomness in material properties as well as uncertainties in the life prediction models; a stress scatter factor; the probability of detection of the crack during an in-service nondestructive inspection (NDI); and uncertainty in the time of the NDI. Supplementary algorithms consider uncertainties in the shape and orientation of a material anomaly at which a fatigue crack will form [10].

The probabilistic calculations can be performed using full Monte Carlo simulation, which always converges to the correct answer given enough simulations (but can be extremely time-consuming), or a tailored Importance Sampling algorithm [11] that can significantly reduce the computational cost with no significant loss of accuracy. The probabilistic calculations can determine not only the probability of failure with and without considering NDI, but also identify the relative contribution to risk from the different regions of the component.

For components in which fatigue cracks can form anywhere in the volume due to the potential occurrence of material anomalies (such as rogue inclusions), a zone-based scheme is used to calculate the probability of fracture [12]. In the simplest case, the probability of component fracture for a single zone is the product of the probability that a material anomaly occurs in the volume of that zone, times the probability of fracture given that an anomaly is present at the minimum-life location within that zone. The total probability of component fracture is then the
sum of all the zone probabilities. As the number of zones increases (the zone
discretization becomes progressively finer), the algorithm converges from above
to the exact answer [13].

The construction of zones, and the refinement of zones in search of algorithm
convergence, is again a manual analysis process made significantly easier by a
highly interactive GUI. However, this manual process could itself be automated,
and this is the logical next step to take in the development of an enhanced
computational framework. Given the availability of automatic fracture model
algorithms that can return a FCG life for any initial crack size at any location in
an FE model (described earlier), automatic risk assessment algorithms can be
derived to progressively refine a zone breakup or, alternatively, perform a
zoneless numerical integration of risk over the volume of the component [14].
This work is pending at this writing.

6. Concluding Remarks

The development of this new computational framework has been continuing since
1995 with support from the Federal Aviation Administration (FAA) through a
series of grants [15, 16]. The framework has been implemented in the DARWIN
(Design Assessment of Reliability With INspection) software, and new advances
are being introduced in new versions of DARWIN as they are developed and
tested. DARWIN development and validation is being conducted in close
conjunction with several gas turbine engine companies, many of whom have also
adopted the software for production use as part of their company design systems.
In short, therefore, the new computational framework for FCG analysis of
components described in this short paper is not merely a theoretical idea or a
research project, but it is in fact becoming the new reality for how practical
damage tolerance analysis of actual hardware can be performed in industry. The
new framework offers significant improvements in accuracy and efficiency and
therefore significant reductions in the total cost of the analysis process.

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8. References

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